13. Gravitational Fields

13.3 Gravitational field of a point mass

Paper 4

Question Paper

1 **(b)** A satellite X, of mass M, orbits a planet at a constant distance $4R$ from the centre of the planet, as shown in Fig. 1.1.

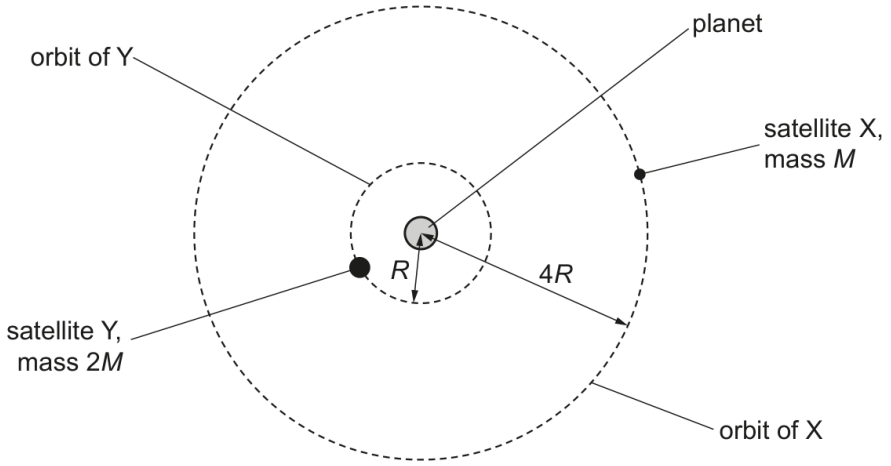


Fig. 1.1 (not to scale)

A second satellite Y, of mass $2M$, orbits the planet with orbital radius R.

The gravitational potential at X due to the planet is $-\Phi$. The planet is a uniform sphere.

(iii) Complete Table 1.1 by giving expressions, in terms of some or all of M, R and Φ, for the quantities indicated for each of the satellites X and Y.

Table 1.1

	satellite X	satellite Y
gravitational field strength at satellite due to planet		
gravitational potential energy of satellite		

[4]

2 The Sun may be considered as a uniform sphere with a mass of 1.99×10^{30} kg and a surface temperature of 5780 K.

A probe with a mass of 2.63 kg moves in a straight line towards the Sun.
When it is at a distance x from the centre of the Sun, the probe measures the gravitational field strength g due to the Sun and the radiant flux intensity F of radiation from the Sun.

(b) For the position of the probe where $x = 1.47 \times 10^{11}$ m:

(i) calculate g

$g =$ ..... N kg^{-1} [2]

3 **(b)** An isolated uniform conducting sphere has mass M and charge Q.
The gravitational field strength at the surface of the sphere is g.
The electric field strength at the surface of the sphere is E.

(i) Show that

$$\frac{M}{Q} = \alpha \frac{g}{E}$$

where α is a constant.

[3]

4 **(a)** **(ii)** Explain, with reference to gravitational field lines, why the gravitational field near the surface of the Earth is approximately constant for small changes in height.

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..... [2]

(b) A large isolated uniform sphere has mass *M* and radius *R*.

Point P lies on a straight line passing through the centre of the sphere, at a variable displacement *x* from the centre, as shown in Fig. 1.1.

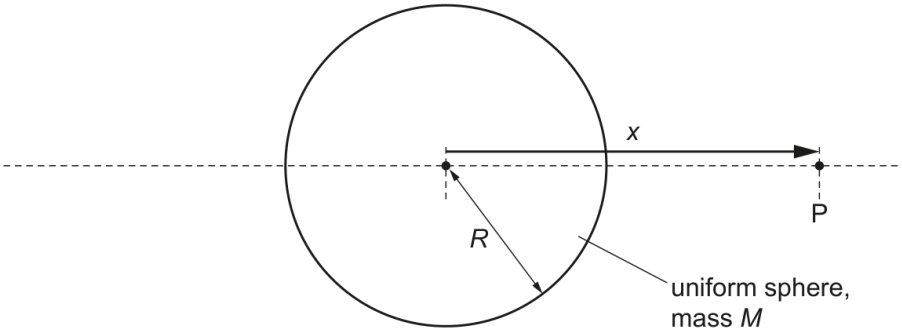


Fig. 1.1

Fig. 1.2 shows the variation with *x* of the gravitational field *g* at point P due to the sphere for the values of *x* for which P is inside the sphere.

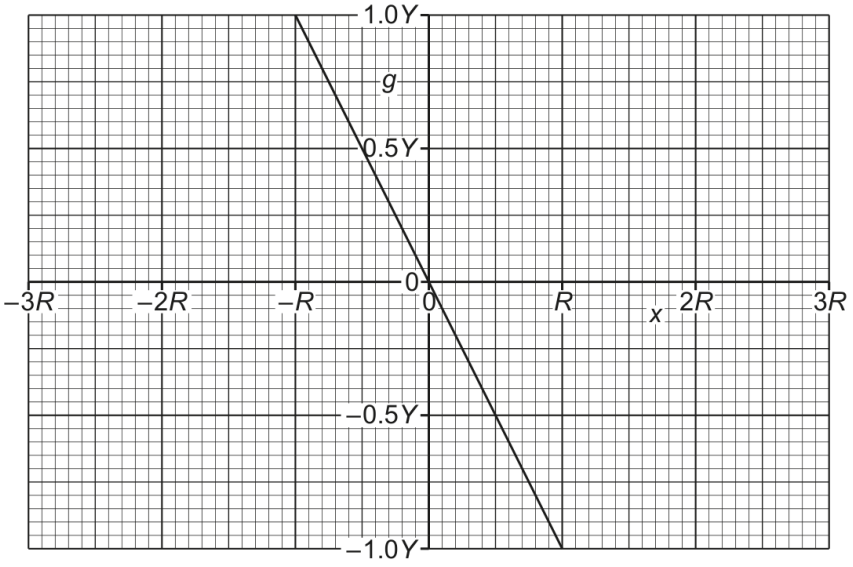


Fig. 1.2

The magnitude of the gravitational field at the surface of the sphere is *Y*.

(i) Determine an expression for Y in terms of M and R. Identify any other symbols that you use.

[2]

(iii) Complete Fig. 1.2 to show the variation of g with x for values of x, up to $\pm 3R$, for which point P is outside the sphere. [3]

5 **(a)** **(ii)** Use Newton's law of gravitation to show that the gravitational field strength g at a distance r away from a point mass M is given by

$$g = \frac{GM}{r^2}.$$

[2]

(b) The Earth has a mass of 5.98×10^{24} kg and a radius of 6.37×10^{6} m.
The Moon has a mass of 7.35×10^{22} kg and a radius of 1.74×10^{6} m.
The Earth and the Moon can both be considered as point masses at their centres. Their centres are a distance of 3.84×10^{8} m apart.

(i) Show that the gravitational field strength at the surface of the Moon due to the mass of the Moon is $1.62\,\text{N kg}^{-1}$.

[1]

(ii) Explain why there is a point X on the line between the centres of the Earth and the Moon where the resultant gravitational field strength due to the Earth and the Moon is zero.

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..... [2]

(iii) Calculate the distance x of point X from the centre of the Moon.

x = ... m [3]

6 **(b)** An isolated planet is a uniform sphere of radius 3.39×10^6 m. Its mass of 6.42×10^{23} kg may be considered to be a point mass concentrated at its centre. The planet rotates about its axis with a period of 24.6 hours.

For an object resting on the surface of the planet at the equator, calculate, to three significant figures:

(i) the gravitational field strength

field strength = ... $N\,kg^{-1}$ [2]

7 **(b)** The Earth E and the Moon M can both be considered as isolated point masses at their centres. The mass of the Earth is 5.98×10^{24} kg and the mass of the Moon is 7.35×10^{22} kg. The Earth and the Moon are separated by a distance of 3.84×10^{8} m, as shown in Fig. 2.1.

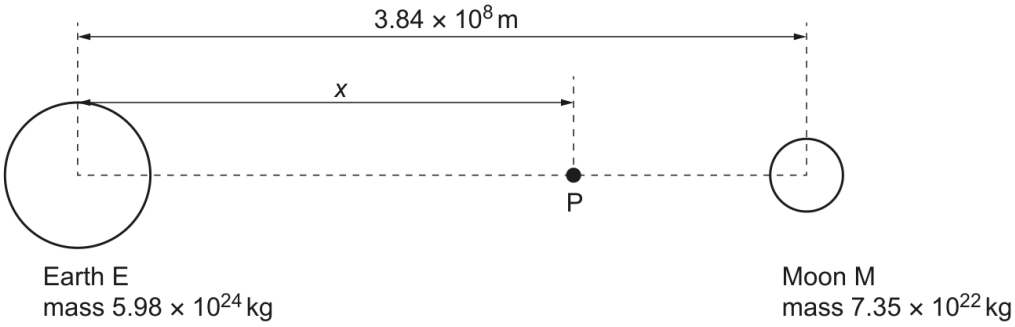


Fig. 2.1 (not to scale)

P is a point, on the line joining the centres of E and M, where the resultant gravitational field strength is zero. Point P is at a distance *x* from the centre of the Earth.

(i) Explain how it is possible for the gravitational field strength to be zero despite the presence of two large masses nearby.

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..... [2]

(ii) Show that *x* is approximately 3.5×10^{8} m.

[2]

8 **(b)** A moon of mass M and radius R orbits a planet of mass $3M$ and radius $2R$. At a particular time, the distance between their centres is D, as shown in Fig. 2.1.

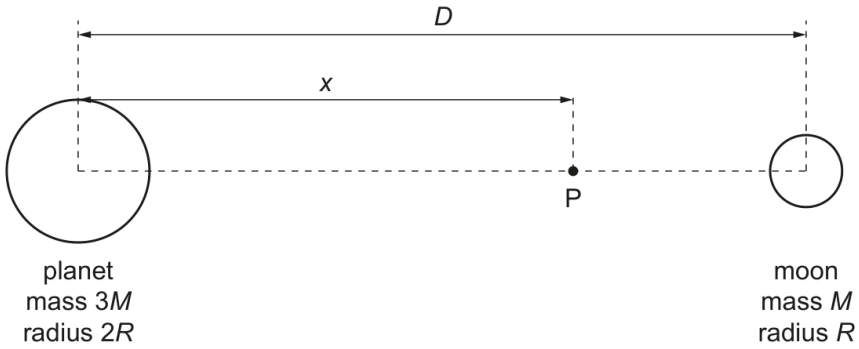


Fig. 2.1

Point P is a point along the line between the centres of the planet and the moon, at a variable distance x from the centre of the planet.

The variation with x of the gravitational potential ϕ at point P, for points between the planet and the moon, is shown in Fig. 2.2.

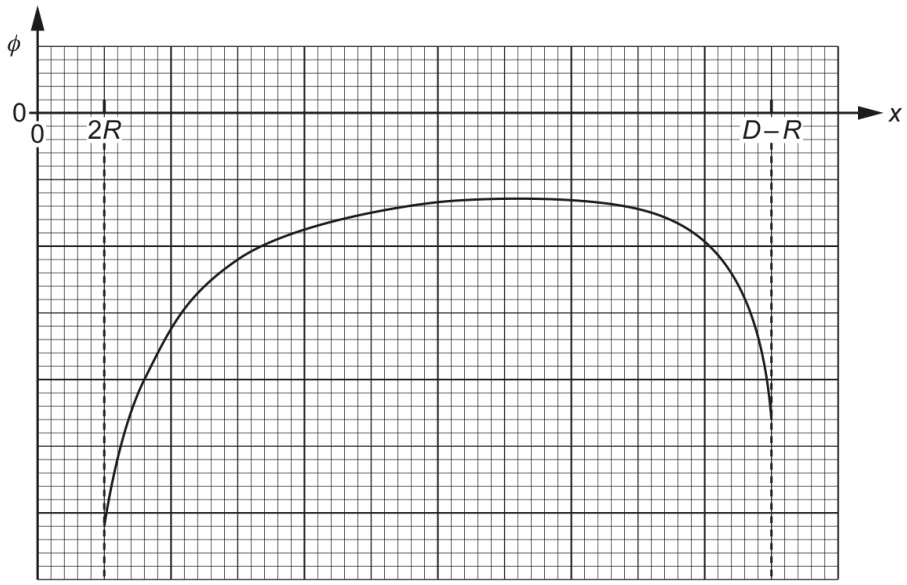


Fig. 2.2

(iii) On Fig. 2.3, sketch the variation with x of the gravitational field strength g at point P between $x = 2R$ and $x = D - R$.

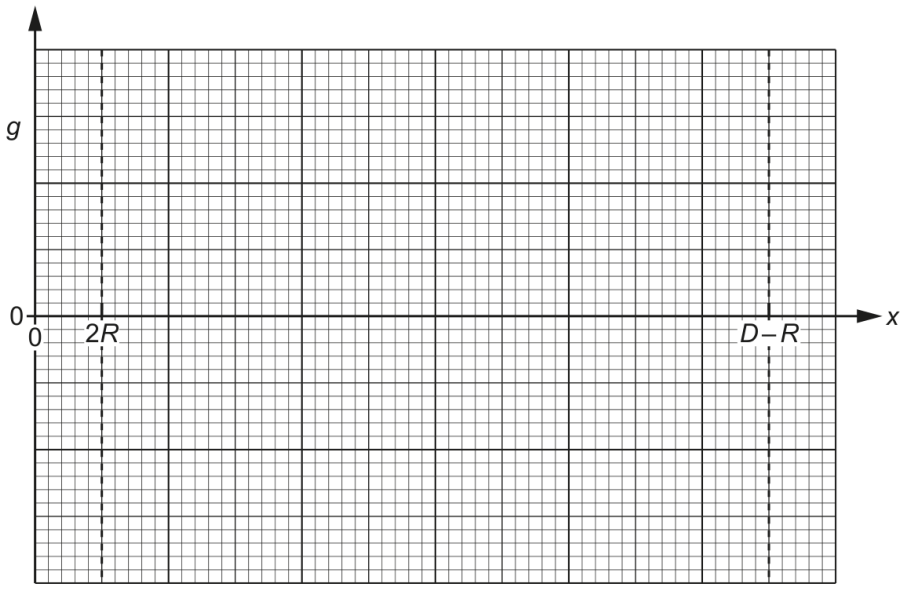


Fig. 2.3

[3]

9 **(b)** An isolated planet may be assumed to be a uniform sphere of radius 3.39×10^6 m with its mass of 6.42×10^{23} kg concentrated at its centre.

Calculate the gravitational field strength at the surface of the planet.

field strength = ... $N\,kg^{-1}$ [3]

(c) Calculate the height above the surface of the planet in **(b)** at which the gravitational field strength is 1.0% less than its value at the surface of the planet.

height = ... m [3]

10 (a) (ii) By reference to lines of gravitational force near to the surface of the Earth, explain why the gravitational field strength *g* close to the Earth's surface is approximately constant.

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.....[3]

11 (b) An isolated uniform spherical planet has radius R.
The acceleration of free fall at the surface of the planet is g.

On Fig. 1.1, sketch a graph to show the variation of the acceleration of free fall with distance x from the centre of the planet for values of x in the range $x = R$ to $x = 4R$.

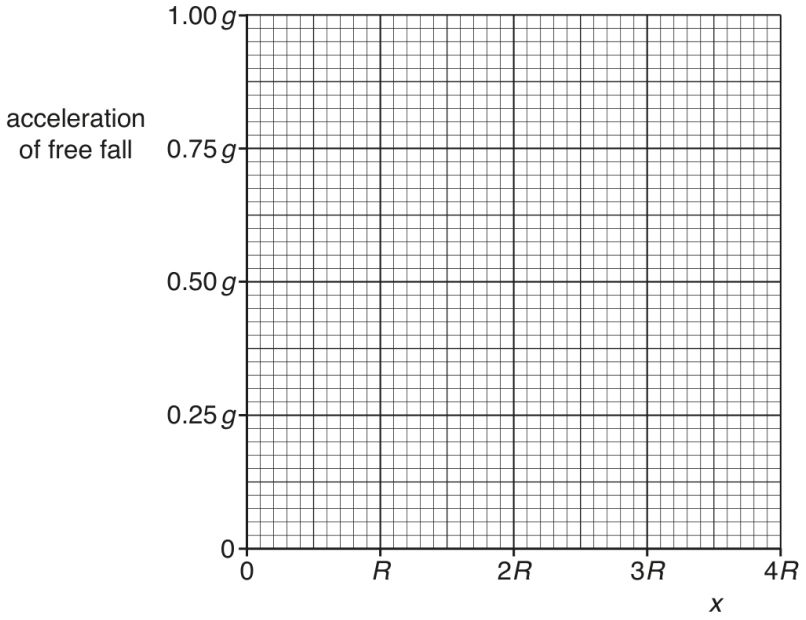


Fig. 1.1

[3]

(c) The planet in **(b)** has radius R equal to 3.4×10^3 km and mean density 4.0×10^3 kg m^{-3}.

Calculate the acceleration of free fall at a height R above its surface.

acceleration of free fall = ... m s^{-2} [3]